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Planetary Position-Velocity Ephemerides Obtained by Special Perturbations

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Position-velocity ephemerides of Venus and the Earth-moon system have been generated using special perturbation methods in which conditions at initial epoch are determined so that the subsequent positions are in best agreement with the Newcomb ephemerides in the least-squares sense. The ephemerides so obtained made possible the 1961 Jet Propulsion Laboratory (JPL) radar observations of Venus and were used in the subsequent determination of the published JPL value of the astronomical unit. They are also being used in generating acquisition ephemerides for future radar observations of Venus and in calculating preflight Mariner standard trajectories. Comparison of the residuals between the Newcomb and the new ephemerides discloses clearly the major discrepancies in the Newcomb theory. The success of the method has led to the current development of an ephemeris library system that will be used to develop internally consistent position-velocity ephemerides of all the planets and the moon and that will be of greatest possible accuracy over long arcs.

AUTHOR

Introduction

THE accuracy to which one can determine the position and velocity of a space probe relative to its lunar or planetary target is limited by the accuracy with which the geocentric position and velocity of the target itself are known. Similarly, the accuracy to which residuals in radar range and Doppler measurements of the planets can be computed depends on the accuracy of the position-velocity ephemeris of the target planet. These are only two examples illustrating the recent requirement for accurate planetary and lunar velocity predictions.

The best sources of position predictions remain the classical planetary and lunar theories, e.g., Brown's improved lunar theory,[1] Newcomb's theories of Venus and the Earth-moon system,[2] and the Hansen theory of Mars as developed by Clemence.[3] These theories yield position predictions in the form of expansions in trigonometric series with time-dependent arguments, the coefficients in the expansion having been obtained analytically as functions of certain constants or "mean" elements. These elements were, in turn, determined by fitting past optical observations in the least-squares sense.

Although numerical differentiation of position tabulations obtained from the source theories is a simple and widely used method of obtaining velocity predictions, it is not sufficiently accurate for the examples mentioned in the foregoing. The major difficulty is that a number of short-period terms with small coefficients have been discarded from the position expansions; however, many of these terms become significant for velocity prediction. In addition, computational and manipulative errors have been discovered (and, in fact, are still being discovered); although known errors have been patched up by empirical adjustments of the mean elements and of the coefficients in the expansions, the effect on velocity prediction is severe. Finally, the published tables have been prepared using numerical methods of limited precision, so that the figures are significant to less than seven decimal places. These roundoff errors are amplified in the numerical differentiation process.

On the other hand, special perturbation methods can be used for generating both position and velocity predictions of a planetary or lunar orbit. There are two difficulties:

1) The accumulation of roundoff and discretization error in the numerical integration eventually destroys the accuracy of the predictions.

2) The numerical integrations can be performed only if initial values (for example, position and velocity components) are specified for some initial epoch.

Once the time interval over which the special perturbation solution is to be valid is specified, the accumulation of error can be controlled by taking a sufficiently small integration step size and by using sufficient precision in the arithmetic computations. Thus, the first objection is essentially only a financial one, and, with the present state of the art of computing machinery, the computing expense is not severe.

The second difficulty is removed by choosing the initial values so that the subsequent *position* predictions obtained by the special perturbations method are the best least-squares fit to the source position predictions over the arc of integration. This yields, of course, a classical orbit determination problem in which the "observations" are, in turn, predictions fitted to the actual observations.

This paper gives a somewhat more detailed description of the technique, presents the results of the work done so far, describes how these results have been used, and discusses the extension currently being developed. Finally, some arguments concerning the use of this technique are summarized.

Description of the Method

Let the problem be to develop a heliocentric position-velocity ephemeris of a planet P of mass m over the time interval $t_0 \leq t \leq t_f$. Available are heliocentric rectangular position ephemerides (x_e, y_e, z_e) of P and of all the other planets covering the interval (t_0, t_f). The heliocentric equations of motion of P are

$$\frac{d^2x}{dt^2} = -\frac{k^2(1+m)}{r^3} - k^2 \sum_i m_i \left(\frac{x - x_i}{\Delta_i^3} + \frac{x_i}{r_i^3}\right) \qquad x \to y, z \quad (1)$$

where (x_i, y_i, z_i) are the tabulated coordinates of a disturbing planet P_i of mass m_i, r and r_i are the radius vectors of P and P_i, and Δ_i is the distance between P and P_i.

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The special perturbation method yields values $x(t,c)$, $y(t,c)$, and $z(t,c)$ of the coordinates (and of the velocity components $\dot{x}$, $\dot{y}$, $\dot{z}$ as well) at discrete epochs $t_0 < t_1 < t_2 < \ldots < t_n \leq t_f$. In general, the epochs are equally spaced, but this is not important. The computed coordinates depend on some parameter set $c = (c_1, c_2, \ldots, c_6)$ which serves to fix the values of the coordinates and velocity components at t_0. Denote $x(t_j,c) = x_j(c)$ and $x_e(t_j) = x_{ej}$, $x \rightarrow y, z$, and define

$$S = \sum_{j=0}^{n} \{[x_{ej} - x_j(c)]^2 + [y_{ej} - y_{ij}(c)]^2 + [z_{ej} - z_j(c)]^2\}$$

Then $c_1, \ldots, c_6$ are to be chosen so that S is minimum, and the tabulation of x_j, y_j, z_j, $\dot{x}_j$, $\dot{y}_j$, $\dot{z}_j$ for these values of the c_i forms the desired ephemeris.

The usual method of minimizing S is to calculate the partial derivatives $\partial x/\partial c_i$, $\partial y/\partial c_i$, and $\partial z/\partial c_i$, $i = 1, \ldots, 6$ for assumed values of the c_i and at the points t_j, and to solve the normal equations

$$\sum_{k=1}^{6} \left[\sum_{j=0}^{n} \left(\frac{\partial x_j}{\partial c_i}\frac{\partial x_j}{\partial c_k} + \frac{\partial y_j}{\partial c_i}\frac{\partial y_j}{\partial c_k} + \frac{\partial z_j}{\partial c_i}\frac{\partial z_j}{\partial c_k}\right)\right] dc_k = \sum_{j=0}^{n} \left(\frac{\partial x_j}{\partial c_i}(x_{ej} - x_j) + \frac{\partial y_j}{\partial c_i}(y_{ej} - y_j) + \frac{\partial z_j}{\partial c_i}(z_{ej} - z_j)\right)$$

for corrections dc_i to the values of c_i. It is generally necessary to iterate a number of times for convergence.

Any special perturbation method could be used in calculating x, y, z, etc. The authors have used only Cowell's method, since they find it necessary to use double-precision arithmetic (i.e., carrying about 16 decimal figures on an IBM 7090 computer) in order to control accumulation of roundoff error and are reluctant to perform double-precision evaluation of position in a reference two-body orbit. Development of more precise trigonometric routines and acquisition of an IBM 7094 may make the use of the method of Encke or of Herrick feasible.

The choice of the parameters c_i is dictated in part by the numerical integration method used and in part by the way in which partial derivatives of x, y, z with respect to the c_i are computed. In most of their work to date, the authors have approximated these partial derivatives by partial difference quotients, i.e.,

$$\frac{\partial x}{\partial c_1}(t_j, c_1, \ldots, c_6) = \frac{x(t_j, c_1 + \Delta c_1, c_2, \ldots, c_6) - x(t_j, c_1, \ldots, c_6)}{\Delta c_1}$$

and, similarly, for y and z and for $c_2, \ldots, c_6$. However, in our new programs we are reverting to the classical method of ignoring the perturbing planets and calculating partial derivatives directly from two-body formulas (see Ref. 4, p. 241). This latter method requires that the parameters c_i be osculating elliptic elements at the epoch t_0; any parameter set can be used if partial difference quotients are used, and the c_i have been taken to be the position and velocity components at epoch for a Runge-Kutta integration and to be first and second sums at epoch for a second sum or Gauss-Jackson integration.

Calculation of partial derivatives by either method is sufficiently accurate, since they are used only to direct the search, and it is not necessary to go to the more elaborate method of solving the system of variational equations associated with Eq. (1).

The integration step size is chosen so that the accumulation of discretization error is not larger than the expected accumulation of roundoff error. The range of integration is then determined by accumulation of roundoff error. Since this, in turn, depends on the number of steps, it is desirable to employ high-ordered methods in order to permit use of reasonably large step sizes.

Short-Arc Velocity Ephemerides for Venus Radar Observation

The 1961 JPL radar observations of Venus required position and velocity heliocentric ephemerides of Venus and the Earth-moon system and the geocentric position-velocity ephemeris of the moon, both for developing Doppler acquisition ephemerides and for calculating residuals of the radar measurements in the determinations of the astronomical unit.

At first, velocity predictions were obtained by numerical differentiation of the Venus and Earth-moon position predictions computed by Herget directly from Newcomb's tables.[9, 10] These position predictions will be called the Newcomb-Herget ephemerides. It was not possible to achieve Doppler acquisition with these velocity predictions; and even if the experiments could have been performed, residuals calculated from these velocities would have been practically worthless.

The original Newcomb theories were re-evaluated in double-precision, and tabulations of the positions were then fitted over short arcs spanning the period of the observations according to the foregoing scheme. Cowell's method with Runge-Kutta integration at $\frac{1}{2}$-day steps was applied, with components of position and velocity at the beginning epoch used as the parameters c_i and with partial derivatives approximated by partial difference quotients.

The Venus theory was fitted over a 172-day arc. The integration and the source ephemeris agreed to a maximum deviation in any coordinate of 1.71×10^{-7} a.u. and an rms deviation of 1.08×10^{-7}. These residuals are well within the accuracy claimed for the source ephemeris itself.

The heliocentric ephemeris of the Earth (not of the Earth-moon system) was similarly fitted over a 76-day arc to a maximum deviation of 2.42×10^{-7} and an rms deviation of 1.51×10^{-7} a.u.

The velocities so derived completely eliminated the inability to achieve Doppler acquisition, and the experiments were completed successfully, with coherent Doppler observations made over a 60-day arc. The same velocity predictions were then used to calculate residuals of the Doppler observations, and these were in turn reduced, along with range observations, to yield JPL's determination of the astronomical unit. A detailed account of the results is given in Refs. 5 and 6. Note that the high internal consistency and small rms values of the residuals in the observations are due to the excellence of the velocity prediction.

The final value of the astronomical unit was determined from individual estimates obtained by separating the range and Doppler data into blocks, each consisting of observations of one type over a one-day period. The individual values obtained from Doppler data showed a significant trend over the course of the experiment, and the same was true of the values derived from range data. In addition, values obtained from Doppler data were significantly different from values derived from range data. It was determined that these deviations could be greatly reduced by asserting a correction in the mean heliocentric longitude of either Venus or of the Earth-moon. Since the sense of this correction was the same as the sense of the corrections derived by Duncombe of the U. S. Naval Observatory,[7] the Duncombe corrections were applied to the Newcomb theory, the theory re-evaluated and tabulated, the tabulations fitted as in the foregoing, and the residuals calculated again and reduced. Again, the numerical integration positions agreed with the Duncombe ephemeris to about the same accuracy as before.

About half of the significant deviation in values of the astronomical unit as just described was removed. This suggested the possibility of combining the new radar data with the optical observations as reduced by Duncombe in order to derive still more accurate corrections. Such a project has been undertaken at JPL. However, the short arc over which the position was fitted is not long enough to

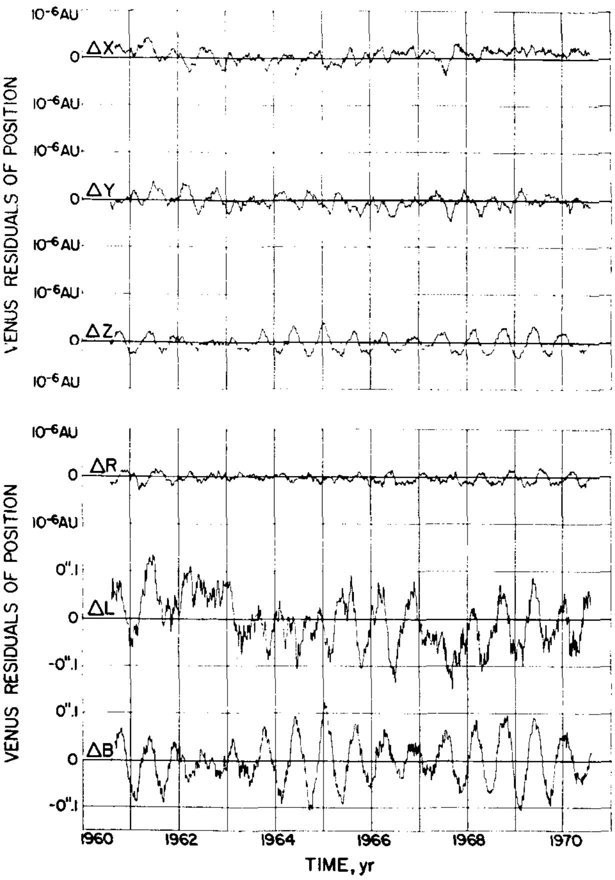

Fig. 1 Venus residuals of position.

insure that the provisional mean elements of Newcomb are mirrored to sufficient accuracy in the integrated ephemeris, and it is necessary to fit over longer arcs as described below.

Extended-Arc Position-Velocity Ephemerides of Venus and the Earth-Moon

A second set of position-velocity ephemerides of Venus and the Earth-moon system was developed, this time with the numerical integration extended to a 10-year arc,‡ from July 1960 to July 1970. Cowell's method with a second-sum integration at two-day steps was used. All calculations were performed in double-precision. Starting values for the numerical integration were obtained from integration formulas similar to the second-sum formula. The first and second sums at epoch were used for the parameters c_i, and partial derivatives were estimated by partial difference quotients.

The major difficulty in the short-arc fits was due to the attempt to fit the Earth alone, rather than the Earth-moon system. The ephemeris actually desired was the geocentric position and velocity of Venus, which is easily obtained from the heliocentric position and velocity of Venus and the Earth-moon system and the geocentric position and velocity of the Earth-moon barycenter. The latter is obtained directly from the geocentric lunar ephemeris (for specified values of the Earth/moon mass ratio and the solar parallax), and, because the values are small, not as many significant figures are required. Thus, numerical differentiation of the tabulated lunar positions gives geocentric lunar and geocentric Earth-moon barycenter velocities to sufficient accuracy. Hindsight also indicated that the re-evaluation of the Newcomb theory was not critical, since the numerical integration and least-squares fitting is itself a smoothing operation. Because it is necessary to work as nearly as possible with the same provisional ephemerides used by Duncombe in order to combine his normal equations with normal equations of the radar observations, only the Newcomb-Herget ephemerides of Venus and the Earth-moon system were fitted.

Plots of the residuals in the sense (Newcomb integration) are presented in Figs. 1 and 2. The maximum residual in any coordinate is 4.76×10^{-7} a.u. for Venus and 8.11×10^{-7} a.u. for the Earth-moon, whereas the rms values are about 2.65×10^{-7} a.u. and 4.32×10^{-7} a.u., respectively. These plots also show clearly the periodic nature of the residuals, with the major period equal to the sidereal period of the body. Because of the known accuracy of the numerical integration process, the authors claim that these residuals measure the inadequacy of the Newcomb-Herget tables. The tables are known to be inadequate in that

1) Certain terms in the latitude included in the theory were omitted in computing the published values.

2) There is a major manipulative error in Newcomb's theory of the Earth-moon, as noted by Clemence.[8]

3) The coefficients in the expansions are given to at most 0.″001 and reflect both roundoff errors and computational liberties taken by Newcomb.

4) Most important of all, the theories are only of first order.

In the process of fitting the integrations to the Newcomb-Herget ephemeris, integration was continued only so long as the deviations $[(x_e - x)^2 + (y_e - y)^2 + (z_e - z)^2]^{1/2}$ remained less than 10^{-5} a.u., and second sums were held fixed at their first guesses until first sums were close to convergence. The initial guesses to the first sums were derived from velocity at epoch estimated by numerical differentiation. These were good enough to permit integration for less than a six-month arc before the tolerance was exceeded. After the first correction to the first sums, the arc of integration could be extended to several years, and, after the second correction, it was possible to consider the entire 10-year arc. The second sums were corrected after the third iteration, and further corrections after the fourth iteration were not actually significant.

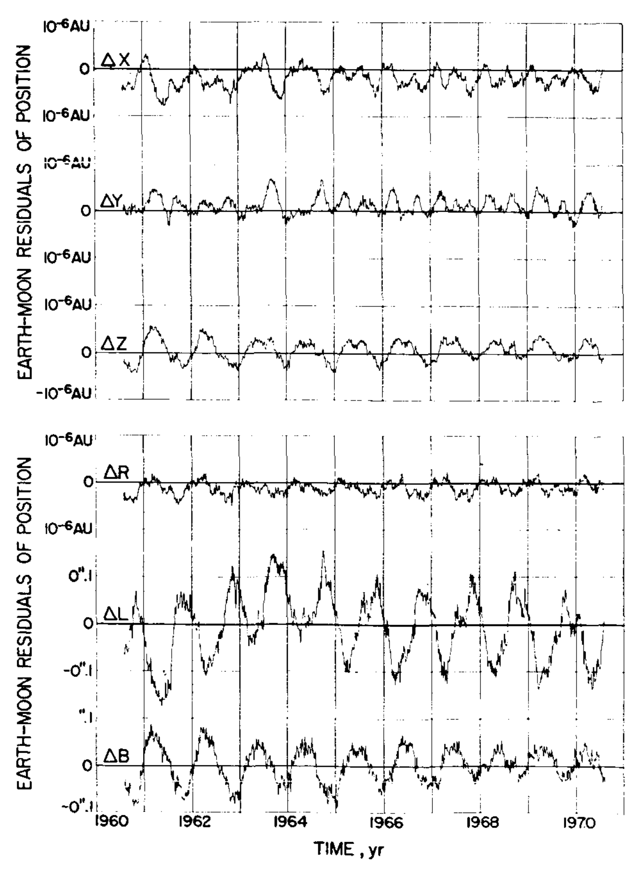

Fig. 2. Earth-moon residuals of position.

‡ More precisely, the arc was 3648.0 days.

The position-velocity ephemeris so obtained has already been used at JPL for a number of purposes, chief among them being the calculation of the 1962 Mariner-Venus pre-flight standard trajectories and the Mariner-Venus orbit determination and data reduction now in process. In connection with this application, it has been demonstrated that a change of center of coordinate system from Earth to sun at any time during the computation of the trajectory of a Mariner vehicle (including no change of phase at all!) yields the same miss distances at Venus within ± 20 km, giving still another verification of the consistency between the position and velocity predictions obtained from the numerical integration.[12]

A second major use of the position-velocity ephemeris has been in developing Doppler and range acquisition ephemerides for the current radar observations of Venus.

Finally, the velocities are being employed for the original problem of obtaining corrections to the mean elements of Venus and the Earth-moon system.

Ephemeris Library System

The preceding results demonstrate the feasibility of the method for deriving velocity predictions consistent with position predictions. We plan to extend the results systematically to all the planets.

The major tool will be an IBM 7090 or 7094 program, called PLOD for planetary orbit determination, which is now being developed. The program employs either Cowell's or Encke's method evaluated in extended precision arithmetic, with partial derivatives calculated analytically. The integration method is second sum, and present plans are to make it capable of integrating either forward or backward.

It is not yet certain for how long an interval of integration accuracy can be maintained, but rough estimates indicate that 40-year arcs for Venus, Earth-moon, and Mars are possible.

A great deal of supplementary effort is involved, the major amount in obtaining the best source position predictions for each of the planets and the moon. This has been accomplished in the case of the moon by a program developed by Block which evaluates Brown's Improved Lunar Theory[11] and for Venus and the Earth-moon system by the previously mentioned program, which evaluates the Newcomb theory. It will be possible to include corrections to the mean elements as derived by Duncombe and as being rederived at JPL for the evaluation. In addition, a program is being written which will evaluate the Hansen-Clemence third-order theory of Mars.

As a result of this project, magnetic tapes as well as tape-reading and tape-editing programs will be available for use in solving trajectory problems. The project is organized on a continuing basis so that, as new and more accurate theories become available, their evaluations will be fitted and placed in the Ephemeris Library.

Summary

Obtaining position-velocity ephemerides by fitting source position predictions has proved eminently satisfactory and is considered the standard technique at JPL. However, the authors anticipate arguments and offer these comments:

1) No claim is made that the positions obtained from the numerical integrations are more accurate than the source predictions themselves. However, these positions are gravitationally consistent over the interval of integration, and this is in no case true of the source data. Moreover, the velocity data are consistent with the position predictions.

2) Localized fits over short arcs will not demonstrate the same high consistency. This is easily seen by noting that perturbations in the initial conditions give rise to a secular perturbation, which becomes apparent only over fairly long arcs. Thus, it is the fact that positions fit relatively well over long arcs rather than exceedingly well over short arcs that makes the method valid.

3) The major effort of the numerical integration is to recover implicitly the short-period terms neglected or erroneously handled in the general perturbation source theory. These terms are particularly important for the velocity prediction.

4) Thus, deriving expansions for velocity predictions similar to the expansions for position in the general perturbation theories is not expected to be competitive because of the much slower convergence anticipated.

5) The intermediate source theories cannot advantageously be eliminated in favor of fitting the numerical integration directly to observations (both optical observations of the past and current radar observations), since these observations extend over time periods too long to be covered conveniently by numerical integration. In fact, a strong case can be made for developing much more accurate general perturbation theories, concentrating most strongly on secular terms, since periodic terms can be re-introduced via the numerical integration. Such theories are being developed at JPL now.

6) Finally, no statistical argument is advanced for using the least-squares criterion, since it is clear that source "errors" are scarcely normal, uncorrelated, or even random. Perhaps a least-uniform approximation would be preferred. However, the added complexity of analysis and computation does not seem justified in view of the excellent results obtained via least squares.

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